To Live and Die in CA

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Abstract— *Biological systems in extreme environments often show patterning, which have not been completely explained. To aid in the explanation of biological patterning, a density based cellular automaton theory is introduced and the implications to biological systems are shown. Maximum region size by density rules are explicitly calculated. Experiments which display the resulting behavior, which is akin to chaos and strange attractors are observed, and large feature sizes are explained by birth and death density parameters. Effects of initial conditions, birth and death rules, radius of calculation, and weighting methods are considered, and the experimental results discussed and compared to actual biological systems.*

Keywords: Cellular Automata, chaos, density rules, biovermiculations

1. Introduction

Patterns in biological systems are not rare. However, some patterns are particularly striking and appear to be the result of more than just the intrinsic growth behavior of individual organisms. For example, we have documented complex mazelike patterns in microbial mats in a variety of environments and over a variety of scales [3], [2] and similar patterns have been noted at even larger scales by other investigators [7], [6], [5], [9], [4].

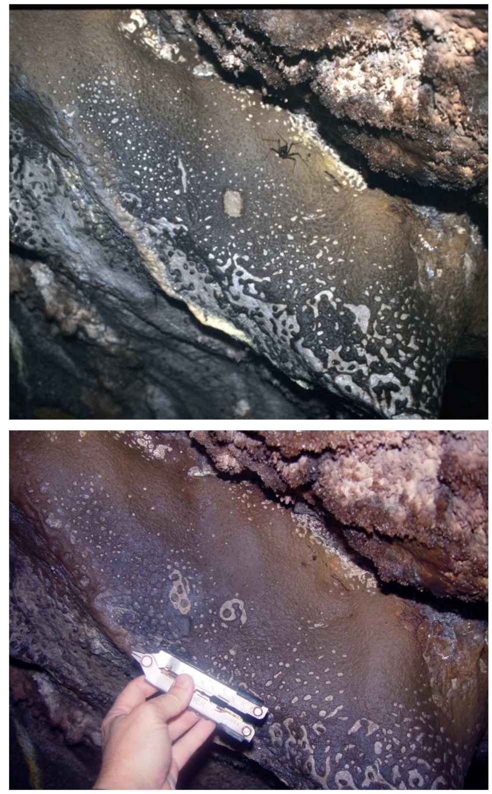
We have focused especially on such patterns in cave environments and to a lesser extent in cryptogamic soil crusts in deserts [1], [8]. The cave examples are particularly amenable to analysis of the complex behavior of microbial maze mats because they are protected from surface weathering. Additionally, in tropical examples, such mats grow and change at easily detectable rates on the order of months to a few years.

Growth patterns in cave walls are not static over time; even where the overall pattern appears constant, there can be detail changes, Figure 1. Modeling such a system should not only produce patterns similar to those that are observed, it should be capable of modeling continuing change in an established pattern.

Differential equation models are capable of this behavior. We have shown [8], [1] that differential equations can be numerically modeled in a cellular automaton (CA); therefore CAs have the same capability to model change as well as pattern. Observed behavior suggests the need to model a pattern which changes over time while retaining gross characteristics such as population densities and feature size, which has at least some of the properties of a strange attractor.

Population density is the most likely candidate for causing oscillation in a CA, since it is the population density that determines the value of the central cell. Defining the death numbers greater than the birth numbers creates the possibility of oscillation between the limits. We can define a distribution of this density that characterizes a given pattern of cells, and since the distribution is patterned, the density

Fig. 1: Regrowth experiment in Cueva de Villa Luz in Tabasco, Mexico by Louise Hose, (a) April 1999, (b) September 2003



(a)

(b)

[1]The authors would like to thank Eric Lindemann and the unending pot of Zzyzx coffee, which made this research possible.

distribution is non-uniform. We get two distinct density distributions - one surrounding live cells and a different distribution surrounding dead cells.

Both dead and live cells are correlated with other cells of the same type, and the entire CA pattern includes voids alternating with high density regions. Dead cells may be near more than one high density region, leading to density values above critical. Where the birth parameter is close to critical this makes it possible for large correlated regions to be born with a density above critical. Actual distributions are sensitive in particular to radius, which defines how near a cell needs to be to live or dead regions to be affected.

We propose that distributions around 0 and 1 cells define sides of a potential well. If there is substantial overlap between the two distributions, particularly if the distribution around the dead cells has significant height in the birth range, oscillations are possible. Less overlap between the distributions may have the effect of a wide, flat potential well and leads to static solutions.

2. Density Theory

Traditionally, CA theory resolves around the number and location of the living neighbors. In this section we begin to develop a different way of analyzing CA utilizing the density, ρ, and the feature ratio, ω. To formalize this, let $N(r)$ be the number of neighboring cells within radius r, in the infinity norm sense, thus $N(r) = (2r+1)^2 - 1$, and $n(r)$ is the sum of the neighboring cells values out to a radius r. In terms of our above parameters, the density is $\rho = \frac{n(r)}{N(r)}$.

One thing that becomes readily apparent in both the calculated and observed patterns is that the features in the pattern are often very different from the radius used by the CA to calculate them. The size of the features is usually consistent and often rounded, so we characterize the features by the radius of a typical circle. We define the relative feature size parameter, $\omega = \frac{r_f}{r}$, where r_f is the radius of the features. For the feature to be stable we must meet the following conditions:

Non-Shrinking	$\rho_{Birth} \notin \{\rho_l, \ldots, \rho_u\}$
Non-Growing	$\rho_u < \rho_{Death}$

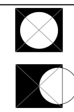
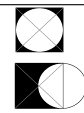
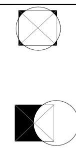
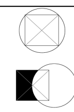
The values of ρ_l and ρ_u are the lower and upper density bounds for a feature, and are characterized in Table 1.

3. Experiments

Four parameters potentially affect the patterning:

1) **Initial pattern** We chose six initial patterns: a horizontal line, a filled square, a hollow square, a corner, a checkerboard, and a random distribution. As long as there was sufficient density for the pattern to grow, and enough time was given, the initial conditions only mattered significantly for radius one systems and slightly for radius two systems.

Table 1: Density bounds as a function of the relative feature size parameter, ω

ω	Geometry	Density
$\omega \leq \frac{1}{2}$		$\rho_l = 1 - \frac{\pi}{4}\omega^2$ $\rho_u = 1 - \frac{\pi}{4}\omega^2$
$1 > \omega > \frac{1}{2}$		$\rho_l = 1 - \frac{\pi}{4}\omega^2$ $\rho_u = 1 - \frac{\pi - \arccos(\omega^{-1} - 1)}{4}\omega^2 - \frac{1}{4}(1-\omega)\sqrt{2\omega - 1}$
$\omega = 1$		$\rho_l = 1 - \frac{\pi}{4}$ $\rho_u = 1 - \frac{\pi}{8}$
$\sqrt{2} > \omega > 1$		$\rho_l = 1 - \frac{\pi}{4}\omega^2 + \omega^2\left(\arccos(\omega) - \sqrt{\omega^2 - 1}\right)$ $\rho_u = \frac{1+\omega-\sqrt{\omega^2-1}}{2} - \frac{(\omega-1)^2\left(\arcsin(\omega^{-1}) - \frac{\sqrt{\omega^2-1}}{\omega^2}\right)}{4}$
$\omega \geq \sqrt{2}$		$\rho_l = 0$ $\rho_u = \frac{1+\omega-\sqrt{\omega^2-1}}{2} - \frac{(\omega-1)^2\left(\arcsin(\omega^{-1}) - \frac{\sqrt{\omega^2-1}}{\omega^2}\right)}{4}$

2) **Birth and Death rules** As indicated by Section 2, we chose the densities such that the birth ranges were less than the death ranges, and tested a wide range of densities for each radius. Using densities for the rules provided a consistency across the otherwise different scales that the radius would normally impose, and allowed us to distinguish two key patterning types that are characteristic of sparse and dense systems. Further, the closer the birth and death ranges were the more chaotic the system appeared.
3) **Radius** (r) We examined radii from one through five, and discovered that unique types of pattern features could be distinguished in systems with radii up to three. For radius 1, patterns were strongly geometric and reflected the starting seed pattern. For radius 2, patterns continued to be geometric and reflect the seed, but chaotic behavior began emerging. Radius 3 patterns showed strong chaotic behavior with an emerging self organization that changes with the death rules. Above radius three we saw no new types of pattern features, though the features were smoother. We have concentrated on radius 3 as results appear closer to the patterned growth we are trying to model.
4) **Weighting method** Each square in the neighborhood

of a cell is multiplied by a weight, such that the sum of the weights is one. For most systems we picked all the weights to be the same (averaging) to be consistent with the density concept. Systems that weighted cells further from the center more heavily tended to be more sensitive to perturbations, while those weighted more heavily to the center should exhibit less chaotic behavior more similar to radius one systems. Non-symmetric weights can cause a system to exhibit more chaotic behavior, including curling.

Examples are computed on a 160*160 grid. The border of the grid is forced to a constant 0 (dead) value to a distance from the edge equal to the radius parameter. Other boundary conditions were considered, but we found from that any distortion caused by this boundary condition is limited to a few radii from the edge.

Initial condition was a random scattering of live cells in a 40*40 region at the center of the grid, adjusted to yield a particular starting population density, generally set midway between the start of the birth range and the critical death density. Most results were generated with 75 iterations. In some cases we ran the system to 140 iterations to verify that oscillatory behavior did not damp out.

Static solutions exhibit varying degrees of ordered patterns, even when the starting conditions are not ordered. The oscillating solutions appear chaotic - similarly shaped and sized features appear on each iteration without repeating any specific form, and similar population densities repeat.

A static case - birth 8, death 12. After 20 iterations, a pattern starts to emerge out of the random initial conditions, Figure 2(a), which is fully static at 75 generations, Figure 2(b). We measured the population density and distributions on the central 40*40 region to avoid edge effects; our final static density was .269, slightly higher than the critical death density of .25 . The population distributions around dead, Figure 3(a) and live cells Figure 3(b) look similar, but the live distribution is narrower and cuts off at the critical density, whereas the dead distribution actually peaks above the critical value.

A chaotic case designed to exhibit oscillations with high density variation - birth 1-24, death 25 and greater. The CA exhibited changes between patterns with similar shapes, but density differences of about 20%. The following pictures correspond to 139 and 140 iterations: Figure 4(a) and Figure 4(b). The distribution graphs for neighborhoods of dead cells, Figure 5(a), and live cells, Figure 5(b) showed large differences in shape, with the dead distribution having a strong peak at 0, lower and mostly flat up to about 20 and then dropping, and the live distribution starting to rise significantly at around 11, peaking just above critical at about 29 and then dropping but not vanishing even at the highest possible value.

Fig. 2: (a) Emergence of static pattern from initial random distribution, (b) Final static pattern, and (c) Mazelike microbial growth on lavatube wall, Kula Kai, Hawaii. Image courtesy of Kenneth Ingham.

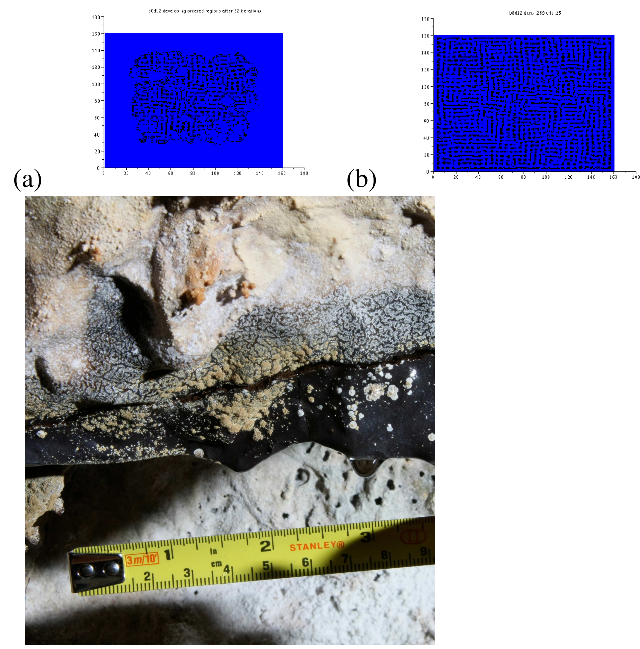


Fig. 3: Population distribution around (a) dead cells, (b) live cells for static pattern

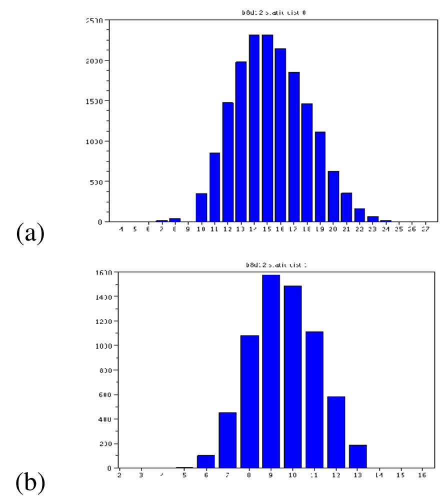
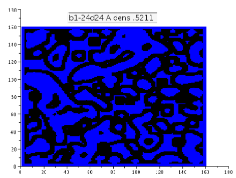
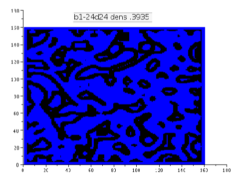


Fig. 4: Successive iterations (a, b) , final development of maximal chaotic case, and (c) Mazelike microbial mat (aka biovermiculation) on limestone cave wall, Cueva de Villa Luz, Tabasco, MX. Image approximately 0.5 m across. Image by P. Boston.

(a)


(b)


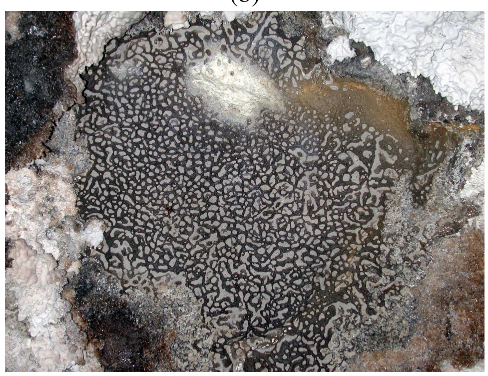
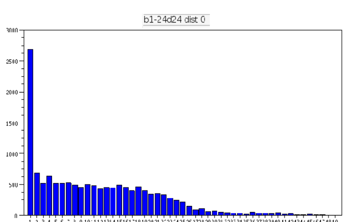
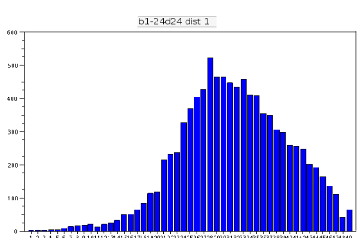
(c)


Fig. 5: Population distribution around (a) dead cells, (b) live cells for maximal chaotic case

(a)


(b)


4. Conclusion

In this work we have introduced CA defined in terms of density parameters. We have observed apparently chaotic oscillation driven by density distributions. We have found mathematical relations between characteristic feature size and shape and the density parameters. The patterns produced by our simple models closely match the patterned growth seen in caves. Furthermore, the changing patterns in the final development of our models closely match the changes over time of biological patterns in the small number of cases available.

We conclude that the concepts and tools here described provide a fruitful approach to the investigation of patterned growth. We are designing new experiments to validate this theory in biological systems, and eventually to better categorize feature size and shape in them so that we can establish changes over time and characteristic time scales. We are investigating the nature of the oscillations to verify whether we have a chaotic or strange attractor. We are also continuing work on extracting the CA rules from photographs of systems, and establishing the correspondence between our density model and differential equation models.

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