An Analytical Model for Network Flow Analysis

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Abstract

We propose to examine the master equation (ME) approach of non-equilibrium statistical mechanics as a model of data communications network flows. The general form of the equation we consider is:

$$\frac{dp_{out}(\ldots)}{dt} = \sum_{in} \Big\{ < \mathbf{n}, f, \mathbf{c}, t, in|\mathbf{TRANS}(in \to out)|\mathbf{n}, f, \mathbf{c}, t, out > p_{in}(t)$$

$$- < \mathbf{n}, f, \mathbf{c}, t, out|\mathbf{TRANS}(out \to in)|\mathbf{n}, f, \mathbf{c}, t, in > p_{out}(t)\Big\}$$

Here, **TRANS** is a *transition* operator that effects the change from one state to another, e.g., *in* to *out*

The ME method has advantages over Monte-Carlo network simulations because it allows for approximate analytical solutions and limiting behavior analysis once a model kernel is chosen. It describes both

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classical as well as quantum information flows within a unified formalism. The ME is more versatile than fluid-dynamical models with no real increase in the computational difficulty of finding a solution compared fluid-dynamical models for situations in which a fluid describes a network flow. For situations in which a fluid model is highly unrealistic, such as when there is no conserved fluid flow and no equation of continuity, a ME can still describe a system; an example is multi-cast flow with concatenation and replication.

Introduction

The issues of the measurement, estimation, and modelling of the performance of a data communications network has a long history. From the early work of Shannon [Sha48] and Hartley to the evolution of software for the simulation of network and protocol performance, there has been a consistent requirement for quantitative methodologies and theoretical frameworks. The best modelling currently available involves the use of stochastic (Monte Carlo) techniques, for example Opnet Modeler [httb] or the NS [htta] package.

Although a stochastic model properly applied can give the most detailed theoretical description of a network, stochastic models as compared with analytic closed-form equations have several disadvantages. For one thing, there are no obvious ways to obtain limiting cases except through repeated simulations and then the application of statistical estimation techniques to the deviates presented. For another, the only method to improve the estimates resulting from a stochastic model is some combination of an increased number of samples and a careful selection of the space from which samples are drawn, the later being done in such a manner as not to bias the result. The former involves more compute time whereas the latter requires a detailed understanding of the underlying distribution which may in fact be the very thing being estimated through the stochastic model.

Hence, we are looking for an analytic formulation to describe the performance of a data communications network. Models based upon fluid dynamics currently are in vogue; however, for reasons that we will discuss below, these models have significant intrinsic limitations. Instead, we have looked at a different formulation based upon time-dependent statistical mechanics, and in particular, the master equation approach.

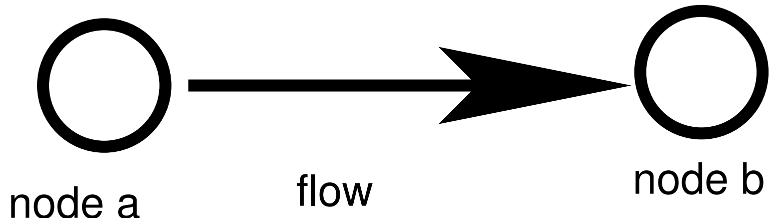

Figure 1: simple single segment information flow

Formulation of the Problem

Our initial model will examine a data communication network one layer at a time, with the result from one layer being used as the input to the next higher layer, following the standard method of encapsulation layer to layer. However, because we are using a time-dependent forumlation, the topology does not need to be static; we can model an adaptive topology as well as a static one.

To describe the network, we need an equation that allows for fundamentally stochastic processes that are time dependent. At present, most networks still use classical information flows based on bits; however, there is in principle no reason to prevent the use of quantum information flows based on qbits. Thus, we seek an equation that can describe both classical and quantum situations.

Such an equation is the master equation from statistical mechanics; one approach to this equation is through the density matrix formalism. Assume that one knows each individual component information flow segment, label this segment

$$|\mathbf{n}, f, \mathbf{c}, t> \tag{1}$$

where

$\mathbf{n}$:= node source and destination index array

f := flow index

$\mathbf{c}$:= vector defining the flow characteristics (the type of flow)

t := time step

Let

$$p(\mathbf{n}, f, \mathbf{c}, t) \tag{2}$$

be the probability (density or point probability) of finding a flow characterized as above; then

$$p(\mathbf{n}, f, \mathbf{c}, t)|\mathbf{n}, f, \mathbf{c}, t >< \mathbf{n}, f, \mathbf{c}, t| \tag{3}$$

is termed an element of the density matrix

To calculate any averaged property of a system, let

$$q(\mathbf{x}) \tag{4}$$

be the operator that represents the averaged property under question then, for some state of the network

$$|\mathbf{S}> \tag{5}$$

the averaged property

$$\bar{q} = \sum\!\!\!\!\!\!\int \delta(\mathbf{x}) q(\mathbf{x}) p(\mathbf{n}, f, \mathbf{c}, t) < \mathbf{S}|\mathbf{n}, f, \mathbf{c}, t >< \mathbf{n}, f, \mathbf{c}, t|\mathbf{S}> \tag{6}$$

is the expected value for the state $\mathbf{S}$; here, $\sum\!\!\!\!\!\!\int$ indicates sum over discrete and integral over continuous, and $\delta\,(\mathbf{x})$ is the appropriate density for the measure being used so that the sum and/or integral converges.
The problem then becomes finding an appropriate representation for the p function for the density matrix that controls whether one has to do integration or summation or both, the density, the measure and any Jacobians that may be needed, the states in density matrix, the state of the system of interest, and the operators q for each characteristic of interest of the system.

To address these question, one issue that needs to be clarified is the exchange statistics of network information flows. Flows are the most natural way to describe data communications over a network in that the desired outcome of a communications transaction is not simply the transmission of a single packet [1] but rather of a meaningful (generally time-ordered) set of packets that contain a useful complete transaction (e.g., a HTTP request followed by a web page displayed to the user).

[1] We use the term *packet* as a generic description for any protocol data unit, irrespective of the name formally assigned to such a unit within a particular standard, e.g., packet, frame, cell, etc.

We also need to address the question of the exchange statistics that describe flows. In quantum mechanics, this concept generally is referred to as the "statistics" of the underlying entities: fermi-dirac versus bose-einstein statistics. For flows of classical information theory, this issue generally is covered by the concept of distinguishability – each classical flow can be distinguished from each other and thus the concept of exchange statistics is not directly applicable. In order to formally unify both classical and quantum concepts, we take a closer examination of the distinguishability of classical flows.

We note that a major flaw with models using ordinary fluid dynamics is that information flows are not conserved in the most restricted sense. For example, in a multicast methodology, one multicast flow can generate numerous end-node unicast flows otherwise the same as the source multicast flow, and thus a typical equation of continuity is not applicable. For this reason, we propose that ***bit flows*** *behave as Grassman numbers and* ***have fermi statistics***, one flow per state, despite the fact that bit flows are distinguishable. In the case of quantum information flows, qbit flows can be fermi, bose, or mixed, depending upon the physical system used to construct the flow.

The question of a method to actually calculate p in the density matrix remains. One such method is to use a Master Equation

$$\frac{dp_{out}(...)}{dt} = \sum_{in} \Big\{ < \mathbf{n}, f, \mathbf{c}, t, in | \mathbf{TRANS}(in \rightarrow out) | \mathbf{n}, f, \mathbf{c}, t, out > p_{in}(t) \quad (7)$$

$$- < \mathbf{n}, f, \mathbf{c}, t, out | \mathbf{TRANS}(out \rightarrow in) | \mathbf{n}, f, \mathbf{c}, t, in > p_{out}(t) \Big\} \quad (8)$$

Here, **TRANS** is a *transition* operator that effects the change from one state to another, e.g., *in* to *out*. To describe both $|state>$ and **TRANS**, we need the time translation operator that moves the system from one time step to the next.

If the system can be described by an "energy" functional $\mathcal{H}$, then the time translation operator formally is $exp(\mathcal{H}t)$, where this $\mathcal{H}$ is for a simple single flow (i.e., one of the states inside the density matrix), and the Master Equation describes the time evolution of the overall system. For this model, we assume classical information flows, $\Delta\Theta(\Delta t)$ as the states (packets) of time interval (length) $\Delta(t)$ where $\Delta\Theta$ is the square wave "bump" $= 1$ for the interval $\Delta(t)$, $= 0$ otherwise.

We assume **TRANS** $= (1 + \frac{x}{t_s})^{-1}$, so that the transition value is 1 for short time steps and falls off for times long compared to the scale time t_s, and that the probability density in the density matrix is Pareto,

$$p(x)dx = dx\frac{\alpha b^{\alpha}}{x^{\alpha+1}} \, , \; x \geq b \tag{9}$$

The use of a distribution such as Pareto is motivated by direct observation of data communications networks using the Internet Protocol Suite. The traffic on such networks is observed to have a self-similarity [WK98] (scale independence) in time for all relevant time scales, and does not obey a Poisson distribution [PF95]. The Pareto distribution has the required "heavy tail" to provide features of self-similarity and is straightforward to parameterize [Arn83]. We note that for the Pareto distribution, $\alpha \leq 2 \Rightarrow var(pareto) = \infty$, $\alpha \leq 1 \Rightarrow mean(pareto) = \infty$, and thus the caveat: the long tail (infinite variance) regime causes some moments to diverge; we must either use cut-off limits or use a smooth cut-off function. Because of the finite capacity and complexity of physical networks, real information flows have such cut-offs naturally imposed by the real system.

As a first test of this approach, we calculated the time delay to flow completion using the above model with a set of smooth cut-off functions, cutting-off at 10, 20, and 30 times the bit-time (in a scale of units measured in bit-times, and thus independent of the actual underlying data rate in terms of the physical reciprocal bit-time) for the longest nominal flow we considered. We used simple three-point Gaussian quadrature to compute the integrals. The results were independent of the cut-off time to less than 5 parts in 100, and the mean time delay times for the three cases were within 1 part in 500.

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