# Constellation Design of a Lunar Global Positioning System Using CubeSats and Chip-Scale Atomic Clocks

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**Abstract—** *Accurate navigation on the moon, Mars, or any other astronomical body is essential to scientific investigation. The research presented in this paper covers the constellation design of a Lunar Global Positioning System (GPS) using the CubeSat platform. Since CubeSats have significantly smaller dimensions than most current satellites, their associated cost is much less to place into orbit. This creates a compelling reason to use them for a Lunar GPS. However, CubeSats require a much smaller atomic clock, which has not been available. Fortunately, there have been recent advancements in chip-scale atomic clocks (CSAC) which can fit within the CubeSat platform. We propose a Rider constellations of two orbital planes and eight satellites per plane for minimum position determination, or fifteen satellites per plane for redundancy at an altitude of* $3.34x10^4 km$. *The CSAC considered is estimated to have an update interval of almost an hour with a ten meter distance error.*

**Keywords:** CubeSat, Chip-Scale Atomic Clock (CSAC), GPS, Lunar, Constellation

## 1. Introduction

Navigation has always been a critical necessity throughout human history. With, the advent of the Global Positioning System (GPS), accurate navigation here on Earth is quickly becoming ubiquitous. As people begin to explore beyond the Earth, navigation will become all the more crucial. Upon the return of people to the moon, navigation will be just as important there as it is here on Earth, not only for exploration on the moon, but also for the astronauts' safety. GPS allows us to determine our position, velocity, and time (PVT) with a high level of accuracy here on Earth. Therefore, a GPS system on the moon would be just as essential of a system. This research was to provide a first look at a CubeSat constellation for such a lunar GPS.

### 1.1 The CubeSat Platform

The CubeSat platform was designed by California Polytechnic State University, San Luis Obispo, and Stanford University [6]. Their purpose was to develop a pico-satellite (a satellite $\leq$ 1kg in weight) platform and delivery system that was affordable and standardized, yet robust enough for other colleges and universities to begin satellite and space research programs [6, 7, 8].

A major constraint and challenge of this research is to keep the hardware to a volume that will fit within the CubeSat architecture. CubeSats are currently designed to dimensions of 10 $cm^3$ and 1 kg payload constraint with the maximum size being three modules [7, 8]. Therefore, the largest volume allowed would be 10 cm x 10 cm x 30 cm and up to 1.33 kg. The compelling reason for considering the CubeSat platform is that it is substantially less costly than current GPS satellites and has already been flight tested on many missions. Currently, a GPS satellite costs roughly on the order of magnitude of hundreds of millions to billions of dollars to develop and deploy [12]. This is due to the cost of the atomic clock, size, and weight of the satellite. CubeSats, on the other hand, are on the order of tens of thousands to a few million dollars [13].

### 1.2 Lunar GPS System Segments

There are three major segments to GPS: space systems, ground control, and the user [1]. First, the space systems segment includes the satellites and their systems. Secondly, the ground control segment includes the ground stations that control, track, and maintain the satellites. Finally, the user segment is the actual GPS receiver and its systems. Another perspective of the segments is that this is the high level design of the GPS system. In this paper, the space systems segment is the only one considered, as the ground control segment and user segment capabilities already exist in many areas.

## 2. Lunar Satellite Constellation

In this section the high level design for the lunar GPS constellation will be shown. Satellite constellations are based upon several factors including the requirements of the system and their orbital parameters. The requirements of the system may include whether constant signal visibility with the satellites are needed. Another could be if worldwide coverage is needed (which it would almost always be). These constraints determine the number of satellites in a constellation and the altitude of their orbits.

## 2.1 General Constellation Design Theory

The general requirements for a GPS constellation design are as follows [1].

1) For PVT determination at least four satellites must be visible at all times anywhere in the world assuming worldwide access is required.
2) The position offsets of the visible satellites need to be such that there pseudoranges with the receiver are as non-singular as possible.
3) The amount of updates from ground based station needs to be kept to a minimum.
4) There needs to be a balance between orbit altitude and transmitter power for the signal.
5) There needs to be a certain level of redundancy in the event of failures.

Considering point 1, one can get away with three satellites if only position determination is necessary. However, considering point 5 as well, the number of visible satellites should be about six [1]. Point 3 is important since if a trajectory needs to be updated, then this requires power and fuel. The fourth is imperative, and additional research will need to be conducted to develop an antenna and transceiver for this subsystem.

## 2.2 The Lunar GPS Constellation Design

Originally, two constellation designs were considered. The first would have been derived from the GPS currently functioning for Earth, specifically the altitude of the satellites. The second was where the constellation and orbital parameters would have been at a lunar synchronous orbit. Although, the orbital altitude of the lunar synchronous orbit would have been ideal, since relativistic errors and the number of required satellites would have been to a minimum, this altitude is too high above the L1 Lagrangian point. This would have caused the satellites to be pulled back by the Earth's gravitational field. Also, even though the altitude for the Earth-based constellation is below the L1 point, it would be too close to the L1 point, causing the circular orbit to perturb, which would have greatly increased the orbital complexity. This phenomena will be discussed shortly since it affects the orbital altitude of the satellites. The proposed GPS constellation design in this paper uses the aforementioned requirements to govern the specific needs for the lunar system, along with global coverage, and inclined circular orbits. Next, several major factors were determined for the constellation's design.

1) The minimum number of satellites to cover the moon.
2) The minimum number of satellites to determine the user's PVT on the moon.
3) The ōptimized orbital parameters for the constellation.
   a) The time it takes for the satellite to orbit around the moon.
   b) The shape of the orbit.
   c) The altitude and inclination.
   d) The number of orbital planes to be used.
4) The signal transmitter power.
5) The optimal level of redundancy.

First, there is a relation between points one and two where one can be thought of as a subset of two. This is generally because the minimum number of satellites to cover the planet is related to how many are visible at a given time from a specific position on the planet. Since this number is usually less than the minimum number of four visible satellites for GPS, then this is why it is a subset. Next, point three modifies the first two, since those parameters are used to determine the minimum number of satellite coverage. Point four affects 3.3 because the more powerful the transmitter, the higher the satellite altitude can be. Finally, point five is important because although to create redundancy one simply needs to place more satellites or planes in orbit, placing too many extra satellite is not only costly, but if the number of satellites is too large, they can cause a singularity to arise in the pseudorange vectors causing errors to grow in the PVT measurements. Therefore, an optimal number of redundant satellites needs to be calculated. For redundancy, usually six visible satellites is deemed satisfactory [1].

## 2.3 Satellite Coverage Determination

Since satellites transmit their signals in concentrated bands of energy, direct line of sight is required for signal acquisition to and from the satellites and the receiver. It is obvious that there are a limited number of visible locations where a receiver can be at a given time with correspondence to the position of a satellite. For instance, a receiver at the south pole does not have line of sight with a satellite in position orbiting above the north pole.

With line of sight being a pivotal requirement, this is used to determine the minimum number of satellites that need to be orbiting within a given orbital plane in order to have line of sight coverage (from here on out referred to simply as "coverage"). The first step that was employed to calculate this minimum number uses Rider's method on determining inclined circular orbits [3]. Consider the first equation for the Rider method:

$$cos(\theta + \alpha) = \frac{cos\alpha}{1 + h/r} \tag{1}$$

In this equation, $\theta$ is the central angle of the body, $\alpha$ is the elevation angle, h is the orbital altitude of the satellites, and r is the radius of the body, in this case the moon. Solving for $\theta$ gives the following equation.

$$\theta = arccos\left(\frac{cos\alpha}{1 + h/r}\right) - \alpha \tag{2}$$

The next step is to use $\theta$ to calculate the minimum number of satellites for a given plane. Below is the next Rider equation:

$$cos\theta = cosc\left(cos\frac{\pi}{s}\right) \tag{3}$$

In this equation, c is a parameter that is defined by Rider as a relation between s and $\theta$ [1]. Then solving for s gives the number of satellites.

$$s = \left\lceil \frac{\pi}{arccos\left(\frac{cos\theta}{cosc}\right)} \right\rceil \tag{4}$$

The reason for taking the ceiling of this equation is to ensure we get an integer value for the satellites since there cannot be a fractional value of a satellite.

Next, the number of satellites for GPS purposes can be determined. First, the orbital altitude of the satellites needs to be determined. In order to do this, Kepler's third law was used [2]:

$$\frac{t^2}{r^3} = \frac{4\pi^2}{Gm} \tag{5}$$

For this equation, t is the orbital period, r is the orbital radius, G is the gravitational constant, and m is the mass of the body being orbited. Lastly, solving for r and subtracting the moon's radius yields the satellite altitude.

$$r = \sqrt[3]{\frac{Gmt^2}{4\pi^2}} \tag{6}$$

$$sat_{alt} = r - r_{moon} \tag{7}$$

# 3. The Lunar Space Segment

The space segment for this GPS is the only segment focused on in this paper. It is worth mentioning that the control segment will be used for sending clock updates for the satellites. There are two major requirements for the space segment. The first is to adhere to the aforementioned constellation design in section 2. The second is to keep the design within the constraints of the CubeSat platform. The major components which contribute to the payload of each satellite would be the electronic and computer hardware, the atomic frequency standard (AFS) clock, the transmitters, the battery, and the solar panels.

During the course of this research it was also found that NASA had discovered that there is an ionosphere in the moon's atmosphere [10]. This discovery dates back as far as the Apollo missions, but had never been qualified until recently by T.J. Stubbs of NASA [10, 11]. It is postulated that the explanation for the lunar ionosphere is from ionized dust particles in the lunar atmosphere [10, 11]. This is important since this ionosphere can have adverse affects on signals sent from orbiting space vehicles down to the lunar surface, producing errors in PVT determination [1]. Assuming the ionosphere is the result of ionized dust particles, it should increase as human exploration expands.

Quite possibly the largest hurdle that needed to be overcame was determining a suitable AFS that would fit within a CubeSat. All GPS satellites use an AFS to ensure a reliable clock frequency to reference. However, these are usually large, heavy, and expensive. Recently, there has been much advancement with this technology, and now there has been developed chip-scale atomic clocks (CSAC) which are about 10 mm$^3$ in volume, and consume only 30 mW [4], making it suitable for an embedded design. In addition, this CSAC has a Allan Deviation less than 1x10$^{-11}$ [4]. Referencing the Galileo GPS specification, and the Allan Deviation the clock validity time can be estimated given a desired distance tolerance [1]:

$$t = \frac{d_{error}}{\sigma_y(\tau)c} \tag{8}$$

In this estimation, $\sigma_y(\tau)$ is the Alan Deviation, c is the speed of light in a vacuum, $d_{error}$ is the allowable distance error, and t is the amount of time that can elapse before a clock update needs to be sent to the satellite from the control segment before the distance error grows past its tolerance.

# 4. Results

To determine the minimum number of satellites for the proposed constellation using the Rider method, a program was made to test various orbital altitudes. The graph below shows the results of the minimum number of satellites for coverage on the moon at these altitudes. These results show

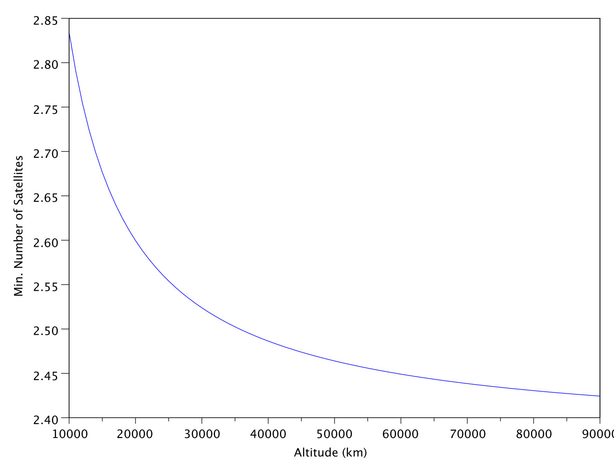


Fig. 1: Minimum Number of Satellites For Moon Coverage Using Rider's Method of Inclined Circular Orbits vs Orbital Altitude

that as the orbital altitude of the satellites is increased, the minimum number of satellites needed to provide coverage exponentially decays. Once this was determined, the desired altitude was calculated to be 3.34x10$^4$km using Kepler's third law. This altitude was determined by choosing an orbital period equal to that of one-fourth of a moon day which equals 6.8305 Earth days. Again, it was desired to have the satellites in a lunar synchronous orbit where they would orbit the moon at the same rate it rotates (27.322 Earth Days), or with an orbital period equal to half of the moon's rotation period similar to that of the Earth's GPS altitude [9]. However, this would have caused the satellites to have an orbital altitude of 8.7x$10^4$ km which is higher than the L1 Lagrangian point (6.3x$10^4$ km) for the synchronous orbit which would have caused the satellites to be pulled in by the Earth's gravity. As for the half moon period orbit, with an altitude of 5.4x$10^4$ km the satellites would have been too close to the L1 point causing the orbits to become unstable. Once the altitude was determined, this was used to determine the minimum number of satellites per plane. Referring back to Figure 1, using the calculated altitude the minimum number of satellites to cover the moon would be approximately 2.5 satellites per plane. Now if this design was simply for having coverage by at least one satellite, then taking the ceiling of this would give us three satellites per plane totaling six satellites. In the aforementioned section GPS is shown to require more satellites. Therefore, a minimum coverage of at least three visible satellites anywhere on the moon calculates to 7.5 satellites per plane totaling 15 satellites (Note: One plane would have one less satellite, or to make it even there could be 16 satellites), and a coverage of at least six satellites would be 15 satellites per plane, with 30 satellites in total.

Next, using equation (8), the time for update was estimated using the data for the CSAC. Assuming a distance tolerance of ten meters, the time for an update would be over 55 minutes and 30 seconds. This time is a little short, but definitely manageable. If the distance tolerance is extended to 50 and 100 meters, the time for update is a little over approximately 4 hours and 30 minutes, and approximately 9 hours and 15 minutes respectively.

## 5. Conclusions and Future Directions

This paper presented a constellation design for a Lunar GPS using the CubeSat platform incorporating CSACs. This GPS considers a Rider constellations of two orbital planes and eight satellites per plane for minimum position determination, or fifteen satellites per plane for redundancy at an altitude of 3.34x$10^4$ km. The CSAC considered is estimated to have an update interval of approximately 55 minutes and 30 seconds for a distance accuracy of 10 m, approximately 4 hours and 30 minutes for a distance accuracy of 50 m, and approximately 9 hours and 15 minutes for a distance accuracy of 100 m. The system is thus feasible, and design costs are well within possible ranges.

Future research will include, but not be limited to optimizing the number of satellites in each plane considering areas of zonal coverage, increasing the time between clock updates, transceiver and antenna design, the possible effects of the lunar ionosphere, and improving error measurement. In addition to this proposed system, differential GPS can be incorporated to enhance PVT accuracy.

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