Depth Approximation in Digital Imaging

There are several methods of approximating the distance between an object and an imaging system collecting the light reflecting from that object. Specifically, we are concerned here with the processing of digitally captured images to discern the perception of object depth. Here we take a look at some of the most common approaches to solving this problem.

Thin Lens Optics:

According to the laws of optics we know that a collimated light beam reflected from an object and passing through a convex lens must converge at the focal point behind the lens. An image plane behind this focal point can collect the light in what is known as the real image.

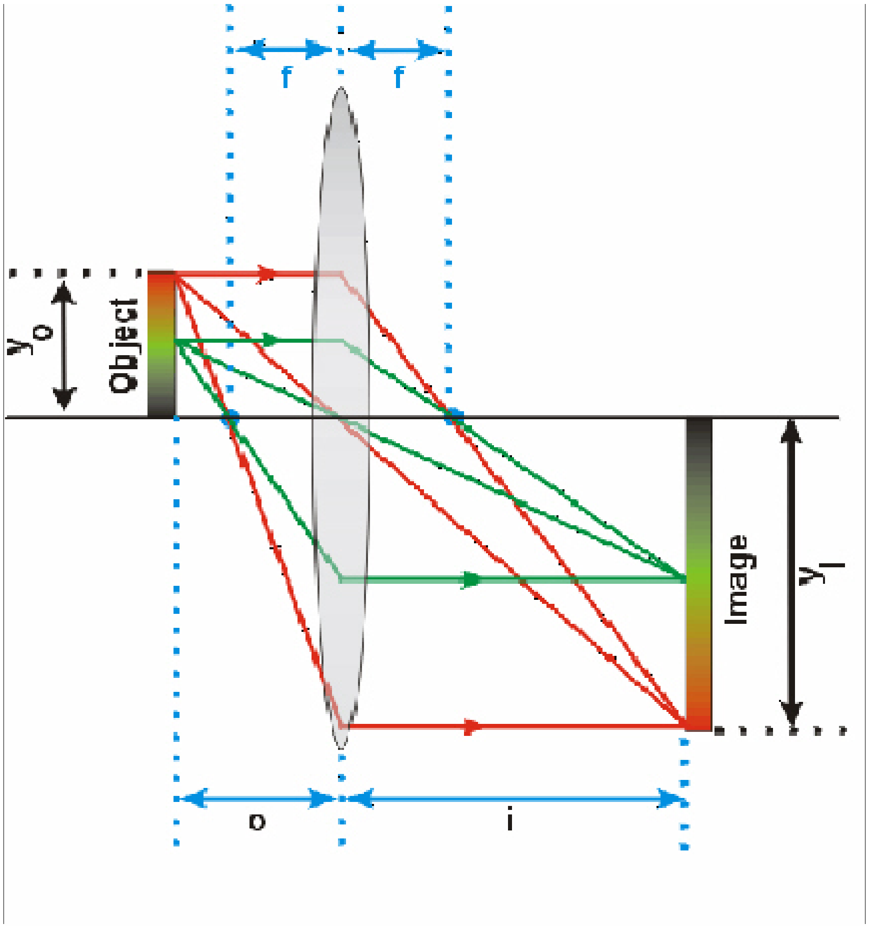


Image from: http://www.upscale.utoronto.ca/IYearLab/Intros/LensOptics/LensOptics.htm

The distances of the object and real image from the lens can be found by the formula:

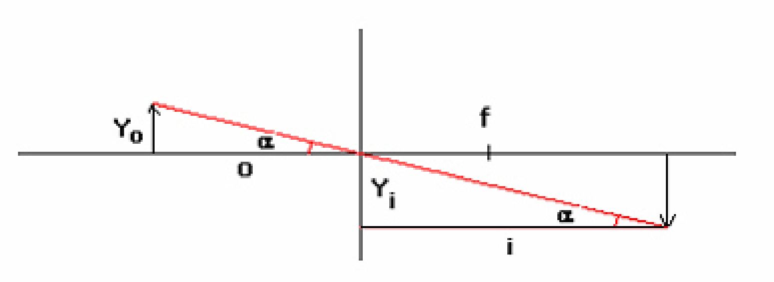
$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

We can also notice that the magnification of the object can be found by the formula:

$$M = \frac{Y_i}{Y_o}$$

Here we can note that when the distances between the object and the real image are equal, the magnification is = 1.

In our case we are interested in a formula that can be used to solve for distance **o** when we know the value of **Yi** and **i** and **Yo**. For this, we can use a specific set of two similar triangles that are formed when the light passes through the center of the lens.



These triangles can be used to solve for **o** as follows:

First we can solve for the angle **α**:

$$\tan \alpha = \frac{Y_i}{i}$$

$$\alpha = \tan^{-1}\left(\frac{Y_i}{i}\right)$$

Now we can substitute and solve for **o**:

$$\tan \alpha = \frac{Y_o}{o}$$

$$o = \frac{Y_o}{\tan \alpha}$$

$$o = \frac{Y_o}{\tan\left[\tan^{-1}\left(\frac{Y_i}{i}\right)\right]}$$

$$o = \frac{i\,Y_o}{Y_i}$$

Example 1 – Finding the depth **o** of an object using thin lens optics:

Object height **Yo**: 10 cm
Distance between image plane and lens **i**: 5 cm
Real image height **Yi**: 3 cm

$$o = \frac{(5cm)(10cm)}{3cm} = 16.67cm$$

Automatic Focus:

When attempting to approximate the distance of an object from the lens, having the object in focus is crucial. In terms of image processing, one of the methods we can use to assure that the desired object is in focus is to move the camera's focus back and forth until we can reach an image with the most contrast possible for the area of interest in the image. As we move the focus, we take an image and sum up the difference in intensity for each adjacent pixel value in the area of interest on the image. This is the amount of contrast of the image object. The larger the sum of the contrast, the less blurred the object and therefore the more focused.

Stereo Vision:

Stereo vision is the application of using two visual systems in some relative position to a 3D object that can be used to estimate the depth or perpendicular distance between the image capture plain and the object. This is achieved through the method of triangulation.

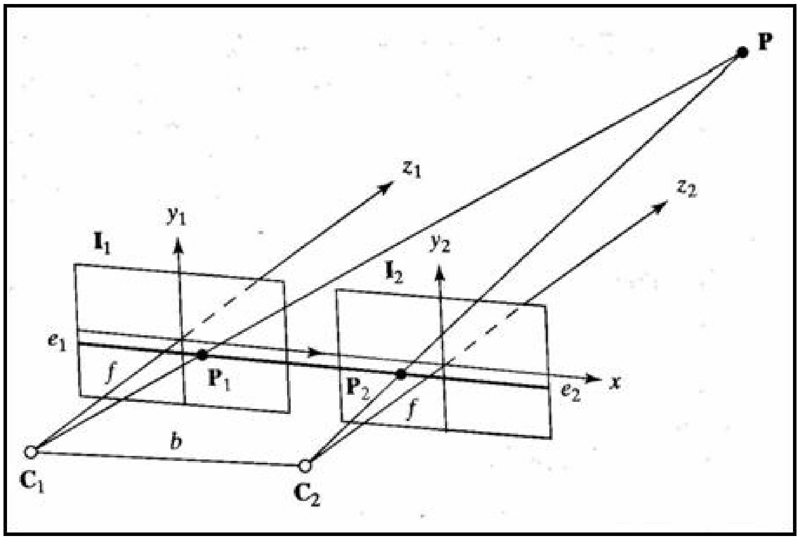


Image from: *http://egweb.mines.edu/tvincent/Welding/fundamentals_of_stereo_computer.htm*

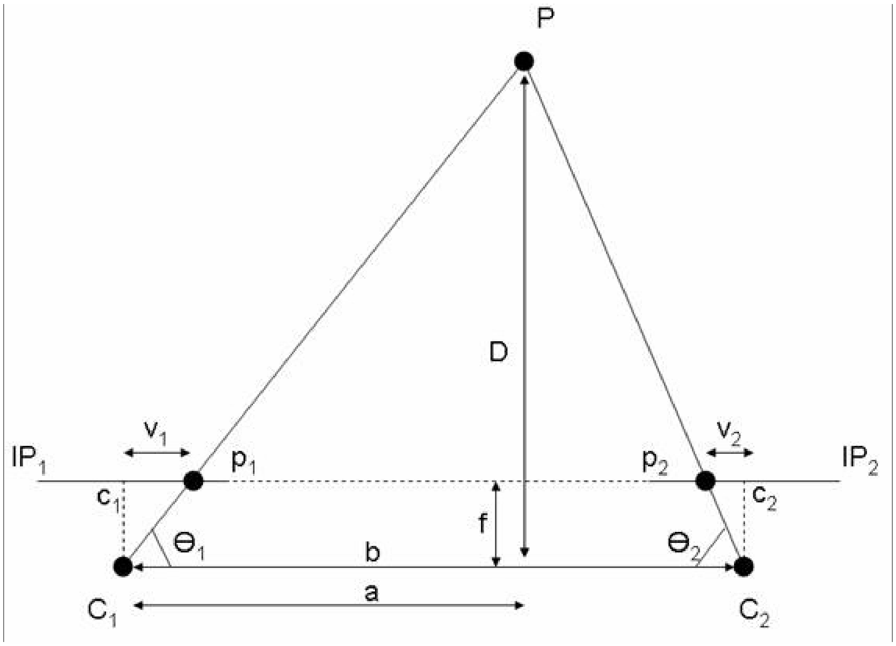


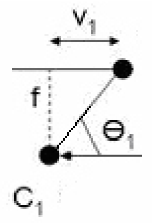
Image from: *http://egweb.mines.edu/tvincent/Welding/fundamentals_of_stereo_computer.htm*

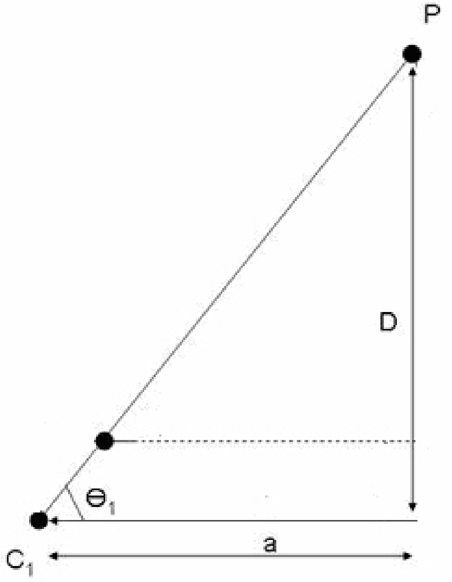
The perpendicular distance **D** between the object and the image capture systems can be found with the formula:

$$D = \frac{bf}{V_1 - V_2}$$

Where:
f = focal length
b = distance between image capture systems
$V_1 - V_2$ = the stereo disparity

Noticing the similar triangles created in the diagram can derive the formula. Using the following 2 triangles from camera 1 we can obtain the proportion:





$$\frac{V_1}{f} = \frac{a}{D}$$

We can do the same using camera 2 triangles and obtain:

$$\frac{V_2}{f} = \frac{a - b}{D}$$

We can now use these two proportions to solve for **D**. First we use one equation to solve for **D** and then we use the other to solve for **a** and substitute.

$$D = \frac{a\,f}{V_1} \qquad\qquad a = \frac{D\,V_2}{f} + b$$

$$D = \frac{\left(\frac{D\,V_2}{f} + b\right)f}{V_1}$$

$$D = \frac{D\,V_2 + b\,f}{V_1}$$

$$D\,V_1 - D\,V_2 = b\,f$$

$$D = \frac{bf}{V_1 - V_2}$$

Example 2 – Finding the perpendicular distance **D** from a stereo camera plane:

b: 15 cm - Distance in the image plane between cameras
f: 3 cm - Focal length of both cameras
V1: 4 cm - Distance of image point from center of image plane 1
V2: 2 cm – Distance of image point from center of image plane 2

$$D = \frac{(15\text{cm})(3\text{cm})}{4\text{cm} - 2\text{cm}} = 22.5\text{cm}$$

References:

http://www.matter.org.uk/tem/lenses/thin_lens_optics.htm
http://en.wikipedia.org/wiki/Lens_(optics)
http://egweb.mines.edu/tvincent/Welding/fundamentals_of_stereo_computer.htm
http://www.cogs.susx.ac.uk/users/davidy/teachvision/vision5.html
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